Name: Mahyar Pirayesh DATE: Nov. 17th, 2023

HW 4.1 Graphing Sine and Cosine Function with Phase Shift, Vertical Expansion and Compression

1. Indicate the transformations for each of the following equations:

a) $y = \sin\theta \rightarrow y = 2\sin\left(\theta + \frac{\pi}{3}\right) - 2$

H.S. $\frac{\pi}{3}$ L $\quad \theta \rightarrow \theta + \frac{\pi}{3}$

V.E. by 2 $\quad y \rightarrow \frac{1}{2}y$

V.S. 2D $\quad y \rightarrow y + 2$

b) $y = \sin\theta \rightarrow y = -3\sin\left(\theta - \frac{\pi}{5}\right) + 4$

H.S. $\frac{\pi}{5}$ R

V.R. & V.E. by 3 $\quad y \rightarrow -\frac{1}{3}y$

V.S. 4U $\quad y \rightarrow y - 4$

c) $y = \sin\theta \rightarrow y = -\sin(-\theta)$

H.R. $\quad \theta \rightarrow -\theta$

V.R. $\quad y \rightarrow -y$

odd function

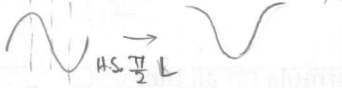
d) $y = \cos\theta \rightarrow y = -\cos(-\theta)$

H.R. $\quad \theta \rightarrow -\theta$

V.R. $\quad y \rightarrow -y$

e) $y = \sin\theta \rightarrow y = \frac{3\cos(\theta)}{2}$

$\cos\theta = \sin\left(\theta + \frac{\pi}{2}\right)$



H.S. $\frac{\pi}{2}$ L

V.E. by $\frac{3}{2}$

f) $y = \cos\theta \rightarrow y = 6 - \sqrt{2}\cos\left(\frac{2\pi}{3} - \theta\right)$

H.R. $\quad \theta \rightarrow -\theta$

H.S. $\frac{2\pi}{3}$ L $\quad \theta \rightarrow \theta + \frac{2\pi}{3}$

V.R. $\quad y \rightarrow -y$

V.E. by $\sqrt{2}$ $\quad y \rightarrow \frac{1}{\sqrt{2}}y$

V.S. 6U $\quad y \rightarrow y - 6$

2. Indicate the amplitude, phase shift, domain and range for each function:

a) $y = 4\sin\theta - 2$

Amplitude: 4

Phase change: 0

D: $x \in \mathbb{R}$

R: $-6 \leq y \leq 2$

b) $y = -\frac{3}{2}\cos\left(\theta + \frac{5\pi}{6}\right)$

Amplitude: $\frac{3}{2}$

Phase shift: $-\frac{5\pi}{6}$

D: $x \in \mathbb{R}$

R: $-\frac{3}{2} \leq y \leq \frac{3}{2}$

c) $y = 3\cos\left(x - \frac{\pi}{3}\right) + 4$

Amplitude: 3

Phase change: $\frac{\pi}{3}$

D: $x \in \mathbb{R}$

R: $1 \leq y \leq 7$

d) $y = 5\cos\left(x - \frac{7\pi}{6}\right) + 4$

Amplitude: 5

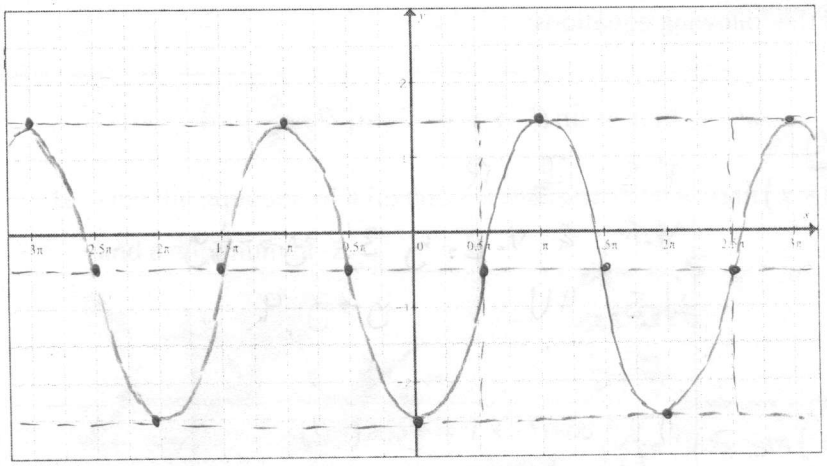
Phase change: $\frac{7\pi}{6}$

D: $x \in \mathbb{R}$

R: $-1 \leq y \leq 9$

$\sin^{-1}\left(\frac{1}{4}\right)$ ↑ IN RAD! Can't use ANGLES!!!

3. Graph the function on the graph provided: $y = 2\sin\left(\theta - \frac{\pi}{2}\right) - 0.5$



Indicate the domain and range:

D: $x \in \mathbb{R}$

R: $-2.5 \leq y \leq 1.5$

$2\pi \approx 6.28$ RAD

Find a general formula for all the x-intercepts

$2\sin\left(\theta - \frac{\pi}{2}\right) - \frac{1}{2} = 0$

$\sin\left(\theta - \frac{\pi}{2}\right) = \frac{1}{4}$

$\theta - \frac{\pi}{2} = \sin^{-1}\left(\frac{1}{4}\right)$

$\theta = \sin^{-1}\left(\frac{1}{4}\right) + \frac{\pi}{2}$

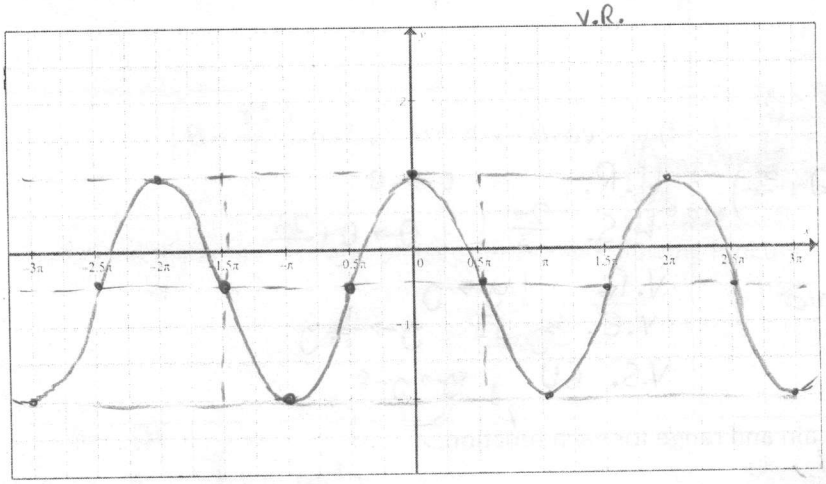
$\theta_1 \approx 1.82$ RAD

$\theta_2 \approx -1.82$ RAD

$(x, y) = (1.82 + 2n\pi, 0), (-1.82 + 2n\pi, 0); n \in \mathbb{Z}$ RAD RAD

4. Graph the function on the graph provided: $y = -\frac{3}{2}\sin\left(\theta + \frac{3\pi}{2}\right) - 0.5$

V.R.



Indicate the domain and range:

D: $x \in \mathbb{R}$

R: $-2 \leq y \leq 1$

Find a general formula for all the x-intercepts

$-\frac{3}{2}\sin\left(\theta + \frac{3\pi}{2}\right) - 0.5 = 0$

$\sin\left(\theta + \frac{3\pi}{2}\right) = -\frac{1}{3}$

$\theta = \sin^{-1}\left(-\frac{1}{3}\right) - \frac{3\pi}{2}$

$\theta_1 = -5.05$ RAD

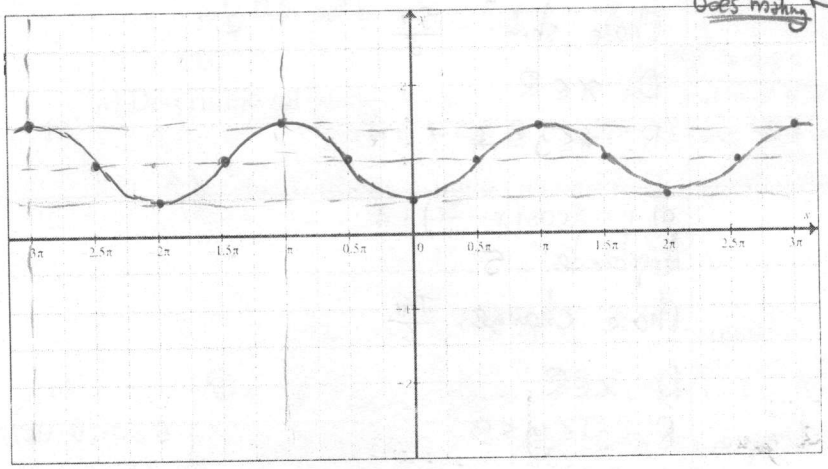
$\theta_2 = 5.05$ RAD

$(x, y) = (5.05 + 2\pi n, 0), (-5.05 + 2\pi n, 0); n \in \mathbb{Z}$

5. Graph the function on the graph provided: $y = 0.5\cos(\pi - \theta) + 1$

② H.R. ① H.S. π L

Does making sense



Indicate the domain and range:

D: $x \in \mathbb{R}$

R: $0.5 \leq y \leq 1.5$

Find a general formula for all the x-intercepts

N/A $\varnothing$

6. Each function in the form of $y = a\sin(\theta - c) + d$, match it with the corresponding graph on the right:

i) $0 < a < 1,\ d > 0$

(g)

ii) $a = -3, c = 2, d = 1$

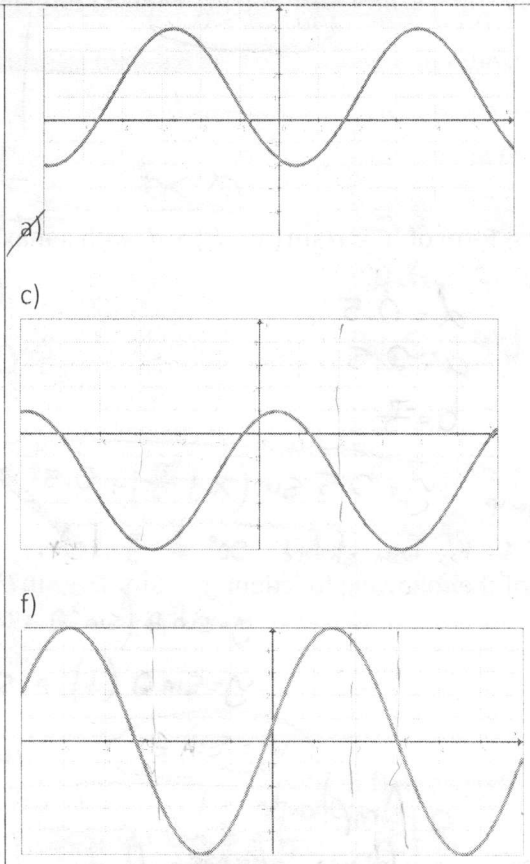
(b)

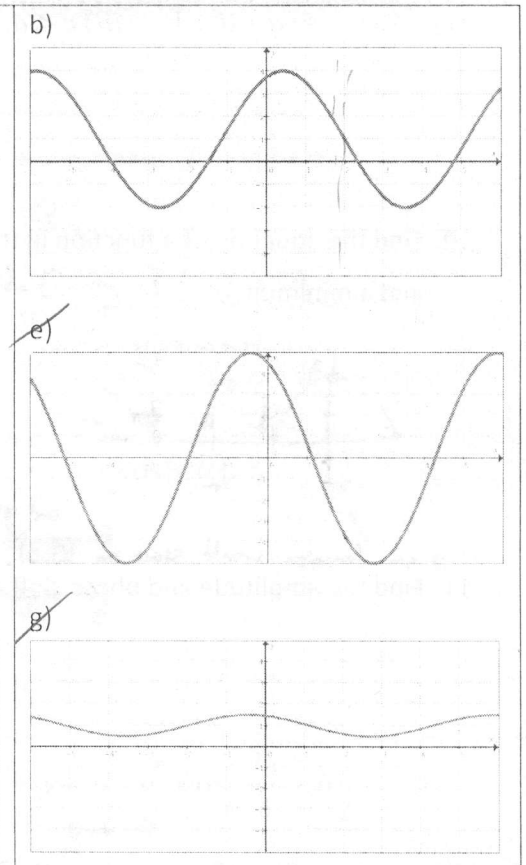
iii) $a > 4,\ c = -2$

(e)

iv) $a < -3,\ c = 3$

(f)





7. Find the equation of a function in the form of $y = a\cos(x - b) + d$ with the following:

a. Maximum at 8, minimum at -2, phase shift of $\frac{\pi}{2}$ units to the right

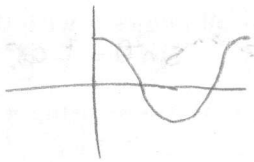
$a = 5$ ↑ Range = 10 ↗ $b = \frac{\pi}{2}$ $d = \frac{8-2}{2} = 3$

$y = 5\cos\left(x - \frac{\pi}{2}\right) + 3$

b. Maximum point $\left(\frac{\pi}{3}, 10\right)$ next minimum point $\left(\frac{4\pi}{3}, 4\right)$

$a = \frac{10-4}{2} = 3$ $d = \frac{10+4}{2} = 7$

$b = \frac{\pi}{3}$ ← shifted from $x = 0 \to x = \frac{\pi}{3}$



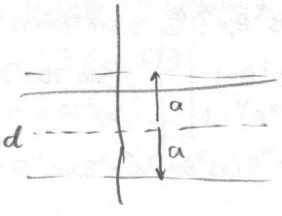
$y = 3\cos\left(x - \frac{\pi}{3}\right) + 7$

8. Given the function $y = a\cos(\theta - b) + d$, what is the range of the function?

Range: $d - a \le y \le d + a$ if $a > 0$

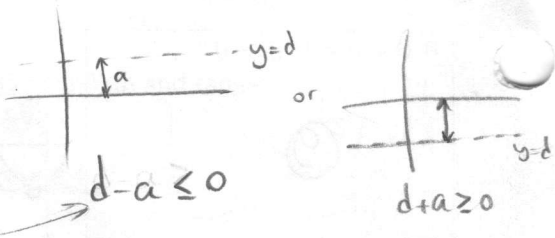
or

$d + a \le y \le d - a$ if $a < 0$



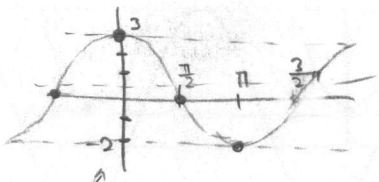
9. In order for the function $y = a\sin(\theta - c) + d$ to have an x-intercept, which of the following must be true?

i) $a < d$ *ii)* $a + d = 1$ *iii)* $c < d$ *iv)* $d > a$ *v)* $a - d = 0$



$a > d$ → $d - a \le 0$; $d + a \ge 0$

10. Find the equation of a function in the form of $y = a\sin(x - b) + d$ with a maximum value of 3 when x=0 and a minimum of -2.



$d = 0.5$

$a = 2.5$

$b = -\frac{\pi}{2}$

$y = 2.5\sin\left(x + \frac{\pi}{2}\right) + 0.5$

a sin function would start at (0,0) and open up. It has been shifted 90° or $\frac{\pi}{2}$ left.

11. Find the amplitude and phase shift of the following function: $y = \sin^3\theta + \sin\theta\cos^2\theta$

$y = \sin\theta\left(\sin^2\theta + \cos^2\theta\right)$

$y = \sin\theta\,(1) = \sin\theta$ → $\sin^2\theta + \cos^2\theta = 1$

$y = \sin\theta$

Amplitude: 1

Phase change: 0

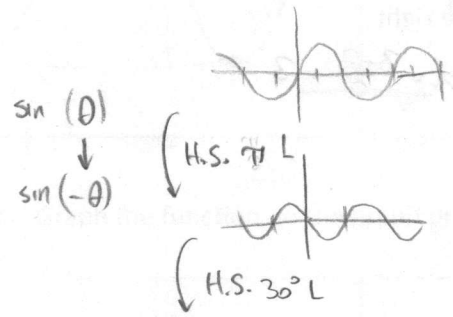
12. Find the amplitude and phase shift of the following function: $y = 4\sin 30°\cos\theta - 4\sin\theta\cos 30° + 3$

$\sin(a-b) = \sin a\cos b - \sin b\cos a$

$a = 30°$

$b = \theta$

$y = 4\left(\sin(30° - \theta)\right) + 3$



Phase change: $-\pi + \frac{\pi}{6} = \frac{-7}{6}\pi$

Amplitude: 4

13. (a) Determine all angles θ with $0° \le \theta \le 180°$ and $\sin^2\theta + 2\cos^2\theta = \frac{7}{4}$.

$\sin^2\theta = 1 - \cos^2\theta \Rightarrow 1 - \cos^2\theta + 2\cos^2\theta = \frac{7}{4} \Rightarrow \cos^2\theta = \frac{3}{4} \Rightarrow \theta = \cos^{-1}\left(\pm\frac{\sqrt{3}}{2}\right)$ neg.

$\theta_1 = 30°$ $\theta_2 = 210°$ $\theta_2 = 330°$

$\theta_3 = 150°$

9. (a) Without using a calculator, determine positive integers m and n for which

$$\sin^6 1° + \sin^6 2° + \sin^6 3° + \cdots + \sin^6 87° + \sin^6 88° + \sin^6 89° = \frac{m}{n}$$

(The sum on the left side of the equation consists of 89 terms of the form $\sin^6 x°$, where x takes each positive integer value from 1 to 89.)

14.

① $\sin(90 - \theta) = \cos\theta$

② $\sin 89° = \cos 1°$

$\sin 88° = \cos 2°$

$\sin 46° = \cos 44°$

③ $\left(\sin^6 1° + \cos^6 1°\right) + \left(\sin^6 2° + \cos^6 2°\right) + \cdots + \left(\sin^6 44° + \cos^6 44°\right) + \sin^6 45°$

Now, sequence ↗

④ $\sin^6\theta + \cos^6\theta = ?$

$\sin^2\theta + \cos^2\theta = 1$

$\left(\sin^2\theta + \cos^2\theta\right)^3 = 1$

$\sin^6\theta + 3\sin^4\theta\cos^2\theta + 3\sin^2\theta\cos^4\theta + \cos^6\theta = 1$

⑤ → $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta\left(\sin^2\theta + \cos^2\theta\right)$

$\sin 2\theta = 2\sin\theta\cos\theta$ → $= \frac{3}{4}\sin^2 2\theta$

$\sin^2 2\theta = 4\sin^2\theta\cos^2\theta$

$= 1 - \frac{3}{4}\sin^2\theta$

⑥ our sequence becomes

$44 - \frac{3}{4}\left(\sin^2 2° + \sin^2 4° + \cdots + \sin^2 88°\right) + \sin^6 45°$

$= 44 - \frac{3}{4}\left(\frac{22}{2}\right) + \sin^6 45°$

⑦

$= 44 - \frac{33}{2} + \left(\frac{\sqrt{2}}{2}\right)^6$

$= 44 - \frac{33}{2} + \frac{1}{8}$

$= \frac{221}{8}$